209 PROBLEM SOLVING AND ACADEMIC SUCCESS

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Tell me and I will forget; show me and I may not remember;
involve me and I will understand. (Confucius)

INTRODUCTION

Problem solving is an almost universal human activity. We begin learning to do it as soon as we begin to interact with our environment as babies. It is part of our exploration of the universe. In spite of its universality however, we frequently approach problem solving in an ad hoc way, often simply applying a learned response or algorithm to a recognized problem type. When this process is carried out, one could legitimately ask if it is “problem-solving” at all. Is it a problem if you already know how to obtain an answer?

So the first question is “What is a problem?” Participants at the Colloque mentioned that problems often had elements of “need” in them: we need a solution, we need to find an objective, and we need to find a way to get to the objective. There was concern that we make a difference between “exercise” and “problem”. We could hypothesize that a problem is a question whose answer you do not know AND which you do not initially know how to determine. A question which does not fulfill both of these requirements is actually an exercise, rather than a problem.

The second question is “Why?”: why should educators teach problem solving? In his discussion of Popperian philosophy in the book “What is this thing called Science?”, A. F. Chalmers (1999) states: “Science starts with problems, problems associated with the explanation of the behaviour of some aspects of the world or universe. Falsifiable hypotheses are proposed by scientists as solutions to a problem. The conjectured hypotheses are then criticized and tested.” Popper is of course referring to the researcher’s encounter with the unknown in the world or universe, while the educator is using problem solving to confront the unknown, as it exists for the student’s own state of knowledge. The educator’s problem solving situation could perhaps be said to be a subset of Popperian problems. This admittedly imperfect parallel may yet allow us to say that if problems are at the origin of science and perhaps of all knowledge, then it would seem that problem solving should have a significant role in our pedagogy.

To quote Frank Jenkins and David Geelan of the University of Alberta, from their presentation at the 21st Annual ChemEd Workshop for the Edmonton Regional Chemistry Council, “Misconceptions in Chemistry”, on May 9, 2003: “Teachers need to provide opportunities for students to actively engage with and test their new knowledge frameworks.” [Jenkins and Geelan’s emphasis] If problem solving addresses questions that are indeed novel problems, then it performs such a function. This activity can thus be a subset of the activity of research scientists exploring the unknown. Therefore, one could postulate that although problem solving is not all that teaching is about, it constitutes one important segment, just as Popperian thinking represents part of what this thing called science may be. The time that is spent in problem solving must not exclude other important aspects of our pedagogy. Any changes that are suggested should respect the time constraints of the teaching year.
To address the question of how problem solving may engage the learner, we need to look at how learning may occur. There seems to be some consensus that we gather incoming information from our five senses via several “intelligence types” (Gardner 1983), which may include “visual”, “auditory”, “kinetic” as well as other intelligences (see Fig. 1). This information first enters our short-term (or working) memory, which is quite limited in capacity (6 or 7 “chunks” usually). These chunks, or units of information, can be simply data or they can be operational instructions that are perceived as single units (Merriam and Caffarella 1991). The most important attribute of a chunk is that it function as a single unit. Long-term learning occurs when the chunks are incorporated into the long-term memory. The manner of incorporation is very important for educators. We would like the chunks to be stored with connections to other items of learning, which some authors call schemata (ibid). If they are simply stored as unconnected bits of information, they may not be available when the learner wishes to call on them at a later date. As shown in Figure 1, there is a two-way communication between the long and short-term memory, and it is this communication that we harness in problem solving.

**Figure 1. Learning and Memory**

**THE PRESENT SITUATION**

It may be helpful to consider the present situation regarding problem solving in Alberta and in Quebec for the discipline of chemistry. A snapshot of the role of problem solving in chemistry education can be obtained by comparing the Chemistry 30 Diploma exam for December 2002 (Alberta Learning 2002) with some winter 2003 exams posted on the Web for Chemistry 101 (Kotovych 2003) and 102 (Apelblat 2003) from the University of Alberta. Table 1 shows the percentage of the exam grade that was devoted to numerical problem solving for Alberta. For comparison, Table 2 shows data from some high school exams in Quebec and some exams given by the author for Chemistry NYA and Chemistry NYB in a Quebec CEGEP.
The CEGEP courses are nearly equivalent to Chemistry 101 and 102 at the University of Alberta, with 85% content overlap, using the same textbook.

In the Alberta high school Diploma exam, 34% of the total grade was devoted to problem solving, while in the first university chemistry course (Chemistry 101), the midterm was 49% problem solving, with the second course final exam (Chemistry 102) being 54% problem solving. There is thus a clear gap in problem-solving expectation between the two levels in that province.

In Quebec, the first semester CEGEP percentage is a bit higher than that of the Alberta high school leaving exam, but second semester escalates considerably, to actually exceed the level of the first-semester University of Alberta course. The numbers do not indicate a clear gap between high school and CEGEP expectations. Of course one must realize that these percentages reveal the priorities of one teacher at one time. Since there are no standardized CEGEP or university exams in either Quebec or Alberta, the percentages will vary from teacher to teacher.

<table>
<thead>
<tr>
<th>Level</th>
<th>Exam</th>
<th>Question Type</th>
<th>Number of Problem Solving Questions</th>
<th>Total Number of Questions</th>
<th>Value of Exam</th>
<th>Percentage of Value of Exam on Problem Solving</th>
</tr>
</thead>
<tbody>
<tr>
<td>High School in Alberta</td>
<td>January 2002 C30 Diploma</td>
<td>Multiple choice, numerical response, and full development</td>
<td>16</td>
<td>58</td>
<td>100</td>
<td>34%</td>
</tr>
<tr>
<td>University of Alberta</td>
<td>Chem 101 midterm</td>
<td>Full development</td>
<td>3</td>
<td>5</td>
<td>43</td>
<td>49%</td>
</tr>
<tr>
<td></td>
<td>Chem 102 final exam</td>
<td></td>
<td>9</td>
<td>12</td>
<td>92</td>
<td>54%</td>
</tr>
</tbody>
</table>

Table 1. Percentage of the Grade Devoted to Problem Solving in Alberta

<table>
<thead>
<tr>
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<th>Value of Exam</th>
<th>Percentage of Value of Exam on Problem Solving</th>
</tr>
</thead>
<tbody>
<tr>
<td>High School in Quebec, Chem 534</td>
<td>June 2002 MEQ Complementary exam</td>
<td>Multiple choice and full development</td>
<td>12</td>
<td>25</td>
<td>100</td>
<td>48%</td>
</tr>
<tr>
<td></td>
<td>January 2002 exam at Quebec High School</td>
<td></td>
<td>10</td>
<td>22</td>
<td>100</td>
<td>34%</td>
</tr>
<tr>
<td>CEGEP in Quebec</td>
<td>Chemistry NYA</td>
<td>Full development</td>
<td>6</td>
<td>15</td>
<td>150</td>
<td>38%</td>
</tr>
<tr>
<td></td>
<td>Chemistry NYB</td>
<td></td>
<td>9</td>
<td>12</td>
<td>150</td>
<td>59%</td>
</tr>
</tbody>
</table>

Table 2. Percentage of the Grade Devoted to Problem Solving in Quebec

The Alberta data would seem to suggest that there may a considerable jump in problem solving from the high school situation to first-semester University of Alberta demands. The question might well be asked, “What effect does that first-semester have on students?” In a personal communication, Margaret-Ann Armour of the Department of Chemistry at the University of Alberta estimates that approximately 20% of the students who begin chemistry drop out before the end of the first semester. In her view, one major difficulty that they encounter is the shift in problem-solving demand. The students have difficulty addressing novel problems in
an independent manner. It would thus seem that teachers of grade twelve chemistry in Alberta have a double challenge:

- to prepare all students for the Diploma exams, and
- to prepare those students who will go on to university-level chemistry for the additional challenge of addressing novel problem situations.

And what about Quebec? Does the fact that the numbers indicate less of a gap between the problem solving demand in high school and CEGEP mean that our students are better prepared for the post-secondary level? As a CEGEP teacher, I would tend to agree with Margaret-Ann Armour that many of our students do have difficulty addressing novel problems in an independent manner. Thus the question is: could it be helpful to consider a slightly different emphasis in the teaching of problem solving?

THE PATHWAY METHOD

This presentation addresses this question by suggesting an approach to problem solving that may remove some of the difficulties that students encounter when facing novel problems (McCalla 2003). The method has its philosophical origins in the work of G. Polya, a mathematician who wrote the book “How to Solve It” (1971). One of his tenets was that a problem is best addressed by examining what it is that the question asks us to find (i.e. the objective) and by working backwards to the information given in the question. The Pathway method presented here involves four elements: Objective, Given, Pathway and Answer. Each element is important, but the Pathway is the distinguishing feature of this method. The Objective and Given provide a very concise summary of all relevant information in the problem. The information on the paper that is in the visual field of the problem solver actually becomes part of the short-term memory, supplementing the usual 6 or 7 chunks (Newell and Simon 1972).

The method of solving problems using Pathways is well adapted to the constructivist approach to learning that has been extensively described in the literature (Gruber and Vonèche 1977). The constructivist perceives learning as a process in which the learners build their own knowledge “through interaction with their environment” (ibid) or perhaps through interaction with the demands of a problem. As the problem solvers reason through the logical connections of the problem, they may become aware of gaps in their understanding, which will require them to revise old knowledge and thus build new knowledge. Vygotsky (1978) suggests that there is a dialogue in learning, with the educator using signs and language to lead the learner into new intellectual development. In the Pathway method, this process finds its expression in the questions that are posed by the existence of a series of objectives that evolve as the questions are answered. Thus in an ideal world, students involved in problem solving would engage in a process of critical reflection, which Schön (1983) and Brookfield (1978) suggest leads to better knowledge construction.

According to Merriam and Caffarella (1991), one important aspect of learning is the creation of “schemata” or patterns, which integrate new knowledge into the already established network of old knowledge, as diagrammed in Figure 1. If the new knowledge does not fit well within the current conceptual framework, the schemata may be restructured. Once integrated, the schemata may be used to create new schemata in which old patterns may be linked to new.

When one uses the Pathway method to address a novel problem, the process of “schemata generation” is fundamental to the development of the Pathway. If the problem solvers already have access to patterns that link the Objective to the Given, the problem is not novel for them. However if the problem is new to them, the Pathway will be creating new patterns, which will have the potential for refining already existing schemata.
Thus, instead of “learning” a pathway (“when you have this kind of problem, this is what you do”), the student will build, step by step, a logical Pathway from the relationships which he or she understands, beginning with the Objective. Each step will be based on a definition, on a chemical, physical or mathematical concept or on an equation expressing an experimentally verified relationship. The reason for beginning with the Objective is that while there may be many items of information in the Given, making it difficult to know which should be used as the starting point in the construction of an answer, there can be only one Objective. It is therefore a sure starting point. For this reason, the Pathway is built backwards, going from the Objective step-by-step back to the Given.

In the Pathway, the student begins the process of logical thought, by writing down the Objective again. The first job is to find something that has a logical connection with the Objective. When such an element comes to mind, it is written down as a new objective and joined to the Objective by a backward arrow:

\[
\text{Objective} \quad \longleftarrow \quad \text{New Objective}
\]

The same process is carried out on the New Objective. The student’s job is to master the concepts sufficiently well to be able to identify the connections, one step at a time. There is no need to solve the whole problem all at once. Once one connection has been made, the student then has a new Objective to seek: he or she once again attempts to find something that has a logical connection with the New Objective.

\[
\text{Objective} \quad \longleftarrow \quad \text{Second Objective} \quad \longleftarrow \quad \text{New Objective}
\]

And thus the creative process continues until the New Objective has a direct connection with the information in the Given. At this point all of the logical analysis is complete.

\[
\text{Objective} \quad \longleftarrow \quad \text{Second Objective} \quad \longleftarrow \quad \text{New Objective} \quad \longleftarrow \quad \text{Information in Given}
\]

Once the Pathway has been determined, only the calculations remain to be done. At this point the problem solver becomes a computer that implements the steps of the Pathway. The student may use dimensional analysis, if the relationships described in the Pathway are either direct or indirect proportionalities, or simply the equations that were incorporated into the Pathway.

An example from everyday life to illustrate how the Pathway method might be used

You need three pounds (lbs) of mushrooms to make a special dish. Mushrooms cost $2.64 a kilogram. You have been told that there are 2.2 pounds in one kilogram. How much money should you take to the store to buy the mushrooms?

**Objective:** $\$

**Given:** Mushrooms @ $2.64/kg mushrooms
2.2 lb/kg
3.0 lb of mushrooms

**Pathway:**

\[
\begin{align*}
\text{Pathway} & : & \text{\$} & \longleftarrow & \frac{\text{kg mushrooms}}{2.64/\text{kg}} & \longleftarrow & \frac{\text{lb mushrooms}}{2.2/\text{lb}} = 3.0
\end{align*}
\]

**Answer:**

\[
\begin{align*}
\text{Answer} & : & \text{\$} = 3.0 \text{ lb mushrooms} \times \frac{1 \text{ kg mushroom}}{2.2 \text{ lb mushroom}} \times \frac{2.64}{\text{kg mushroom}} = 3.60
\end{align*}
\]
An example from chemistry

A chemical analysis requires 2.00 L of 0.150 mol/L AgNO₃. What mass of silver nitrate solid is required to prepare this solution?

**Objective:** \( m \) (AgNO₃) in g

**Given:** 2.00 L of \( \frac{\text{0.150 mol AgNO}_3}{1 \text{ L solution}} \)

**Pathway:**

\[
m = \text{g AgNO}_3 \quad \text{mol AgNO}_3 \quad v \text{ of solution in L} = 2.00
\]

\[
\frac{2169.98 \text{ g}}{1 \text{ mol}} \quad \frac{0.150 \text{ mol}}{1 \text{ L}}
\]

**Answer:**

\[
m \text{ (AgNO}_3) = 2.00 \text{ L solution} \times \frac{0.15 \text{ mol AgNO}_3}{1 \text{ L solution}} \times \frac{169.88 \text{ g AgNO}_3}{1 \text{ mol AgNO}_3} = 5.10 \text{ AgNO}_3
\]

The questioning that would accompany the construction of the Pathway might sound something like the following:

- You want to find the mass (in grams) of silver nitrate. What would you have to know to find grams?
- Now that you have determined that you could use the molar mass of silver nitrate to calculate the mass if you knew the amount (in moles) of silver nitrate, what would you have to know to get the moles?
- Now that you know that you could use the molar concentration to get the moles of silver nitrate if you knew the volume of solution (in L), where could you get the L?
- Oh, you have the liters of solution? Well, then you have finished the pathway!

Obviously, the student who developed this pathway had a good conceptual background, so what happens when the student goes astray during the development of the pathway? The questioning might sound something like the following:

- You want to find the mass (in grams) of silver nitrate. What would you have to know to find grams?
- You have determined that if you knew the volume (in L) of silver nitrate, you could get the grams of silver nitrate, using the density. What is the density of silver nitrate?
- BLOCKED The student realizes that he or she does not have the density of silver nitrate.

At this point the student must return to the original objective and attempt to find another relationship that could give access to the amount (in moles) of silver nitrate. So he or she searches again through the relationships stored in long-term memory. So by a process of elimination of unprofitable pathways, the student eventually comes to one that works.

There are thus at least three rules of Pathway logic that operate to assist the student in finding a correct pathway, aside from the fundamental rule of beginning the reasoning at the Objective.
• Each step must represent a direct relationship: there can be no leaps of logic that involve several relationships that need to be used sequentially.

• When the pathway reaches information that is either in the Given or is known to the problem solver, then the Pathway is complete. The density of water is an example of information that may not be explicitly given in the problem but is assumed to be known to the learner.

• When the Pathway reaches an unattainable, then the problem solver must return to a previous objective, eliminating the branch that was under construction. The definition of “unattainable” varies depending on the problem solving situation. If it is a test situation, then only the information in the Given or widely known relationships like the density of water or the speed of light are available. In an assignment, the student may have access to other sources of information, such as a reference like the Handbook of Chemistry and Physics, so what was unattainable in a test may be attainable in this more open environment.

Once these rules of Pathway logic have been used to obtain a completed pathway, only the calculation remains to be done, which may be accomplished in many different ways, as mentioned earlier.

DOES IT HELP?

One may find the foregoing intellectually appealing without feeling that it is worth pursuing in the classroom. What teachers would like to know is, “Does it make any difference to student success?” As educators we must be conservative in our approach to change. Not every suggested innovation has a positive effect on the learning of our students, and it is our responsibility to protect them from fads that may waste their learning time.

Some research has been done on the efficacy of the Pathway method for first semester CEGEP students (McCalla 2003) which may shed some light on the question of the link with success. Two first-semester general chemistry classes were selected for experimentation. Both groups were taught the Pathway method, but each student had free choice on each problem of whether to use the method or not. Thus for each question, two groups were generated: those students who chose to use the method and those who did not. The Kolmogorov-Smirnov test was used to determine the probability that the two groups were identical. A low probability that the groups are identical, combined with better performance by the students who used the method, would indicate that the Pathway method was beneficial to those students. The details of how this was accomplished are available in the paper, “Problem Solving with Pathways” (ibid).

In the three exams given that semester, there were twenty-seven numerical problems. In fourteen of these, the probability that the groups were identical was very low, while for the other thirteen the probability that the groups were identical was very large. When the results were grouped according to probability, the average grade for the low-probability problems was 5/10, a reflection of the difficulty level of the problems. For these questions, students who used the Pathways were clearly more successful than those who did not. The average grade for the high probability (ie. low difficulty) problems was 8/10, and for these questions there was no difference in performance, whether the students used the Pathway method or not. The results therefore suggest that the Pathway method confers an advantage on students when the problem is more difficult and none at all for easier problems.

The question remained as to whether the results might be explained by the prior performance of the students in their high school chemistry courses. Perhaps only the top students adopted the Pathway method and their greater success was not due to the method, but rather to their intrinsic ability in chemistry. The regression analysis of Pathway use versus high school results, for the students who authorized access to their
records (67% of the total), show rather that there is no correlation between Pathway use and high school grades. One can thus conclude that the method is the likely cause of the greater success on difficult problems.

CONCLUSION

The study reported in the Journal of Chemical Education indicates that the Pathway method benefits students when they are dealing with the more difficult problems, at least at the post-secondary level. The question becomes whether it might not be better to begin addressing problems in this manner at an earlier level. Most students have probably developed habits of thought by the time they reach college or university. Sometimes these habits of thought, which have worked well until that point, become insufficient to the demands of the post-secondary level. Many students have functioned very well by memorizing the problem types that inevitably have appeared on their high school exams. And they have been very successful. But Dr. Armour’s comment that the post-secondary level requires students to become independent problem solvers in novel problems suggests that this new need may be incompatible with the memorize-an-algorithm approach to problem solving. It seems possible that the Pathway method could be taught at the secondary level, to serve two purposes: to prepare students for the demands of university-level science courses by developing their ability to deal with novel problems; and to prepare all students for high school leaving, the contention being that all students can master the step-by-step approach that is used here. The enabling hypothesis behind this proposition is that teachers and students will have to do fewer exercises if the students master the technique of working a pathway from the objective. There will thus be a saving of time that will permit preservation of other aspects of science pedagogy, while improving the development of problem-solving abilities.

No research has yet been done on the Pathway method at the secondary level, but perhaps that is where we should be directing our energies. Focussing on the Objective is a transferable strategy for many types of problem-solving situations. If the habit of thought could be developed at an earlier stage, would we find that problem solving behaviour in all domains would improve?

The author would like to acknowledge with gratitude the gracious welcome accorded by the IONCMASTE discussion group, as well as the support of the Department of Secondary Education of the Faculty of Education at the University of Alberta, during her sabbatical leave in Edmonton. Discussions with Frank Jenkins and David Geelan and many other individuals aided materially in advancing the debate on the role of problem solving in education.

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